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AN ALGORITHM FOR NETWORK DIMENSIONING
UNDER RELIABILITY CONSIDERATION

by

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ABSTRACT

In this paper we introduce a new methodology to adjust link capacities in circuit switched networks taking into account the costing policy and reliability considerations. This methodology, which is an extension of previous work on reliability evaluation using routing models, is based on a cyclic decomposition algorithm which alternates between a routing subproblem and a link capacity adjustment subproblem. The proposed procedure which is shown to converge to a global optimum for the dimensioning/routing problem, has been tested on a 14 undirected arc problem for various levels of link failure probability. The numerical results are extremely satisfactory and they demonstrate the usefulness of the proposed method for proper network dimensioning.

Key words : Network Dimensioning, Telecommunication, Reliability, Circuit Switching, Routing.

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RÉSUMÉ

Nous présentons une nouvelle méthodologie pour l'ajustement de la capacité des liens dans les réseaux à commutation par paquets, en tenant compte de la politique de coûts et de considérations de fiabilité. Cette méthodologie, qui se situe dans le prolongement de travaux antérieurs sur l'évaluation de la fiabilité par des modèles d'acheminement, est basée sur un algorithme de décomposition cyclique qui alterne entre un sous-problème d'acheminement et un sous-problème d'ajustement de la capacité. On démontre la convergence globale de cette procédure qui a été testée sur un réseau non-orienté à 14 arcs pour diverses valeurs des probabilités de pannes des liens. Les résultats numériques sont très satisfaisants et montrent l'utilité de la méthode proposée pour un dimensionnement de réseau adéquat.

Mots-clés : Dimensionnement de réseaux, télécommunication, fiabilité, commutation par circuit, acheminement d'appel.

1. INTRODUCTION

The development of new technologies in the field of telecommunications has forced planners to reconsider the Network Design and Dimensioning Problem. In circuit switched networks, the introduction of non-hierarchical routing has had a profound impact on network planning, specifically on the trunk dimensioning problem. In fact, since the routing policy can be more flexible, the management of the overflow becomes more efficient and, therefore, the dimensioning of the network changes as well (this problem was studied in [1]).

In packet switched networks, the advent of wide-area networking for computers have introduced new significant costs to be taken into account in network planning. For instance, the cost of network delays versus excessive link capacities have forced planners to find new ways of optimizing resources. One way to improve the design of the network has been to consider the design and routing problems simultaneously. In the past, either link capacities were fixed and flows were assigned to optimize some measure of performance (see [2-4]) or the flow was considered to be known and arc capacities were selected according to a cost minimization function (see [5, 6]). The design-routing optimization has been recently explored by Gavish and Newman [7], Gavish [8], Rosenberg [9] and Leblanc and Simmons [10]. Another advanced technology that has drastically changed the design of packet and circuit switched networks is the fiber optic links. Prior to the introduction of fiber optics, the design problem considered the compromising of costs vs. delay in data networks and costs vs. blocking in telephone networks.

Nowadays fiber optic links have such capacity that delay or blocking problems are minimized but a new element is added to the design: reliability. The design of circuit switching networks under reliability constraints has been studied by Gavish et al. [11]; in that paper the problem of establishing node and link capacities to satisfy traffic demands in no-failure and failure conditions was formulated and partially solved. The problem of finding a reliable routing in packet switched networks has been recently attacked by Newman and Gavish [12]. Sanso [13] and Sanso et al. [14] have proposed a measure of reliability in circuit switched networks which is based on the routing policy.

In this paper, we introduce a new methodology to adjust capacity links in circuit switched networks taking into account the routing policy and reliability considerations. This work constitutes an extension of the methodology developed in [13] and [14] to evaluate network reliability using routing models.

2. PRELIMINARIES

In this section we present some preliminaries regarding the routing model and the network performance evaluation measure that will be useful in the description of the Dimensioning and Routing Algorithm (DRA) to be presented later on.

The Routing Model

In [13], several routing models were proposed. The objective of those models was to minimize the network lost call traffic (LCT) under a centralized or decentralized routing policy.

The model that we use in this paper is a centralized optimization model. This model can be formulated as a multicommodity flow problem in which each commodity corresponds to the flow having the same origin.

Traditionally, multicommodity problems have been notoriously difficult to solve due to the presence of rigid capacity constraints. In the routing model under consideration (RM), there are no capacity constraints. In fact, these constraints have been eliminated and replaced by a non-linear penalty function reflecting the stochastic character of the demand. This function represents the network expected lost call traffic, which is, at the same time, a parameter that we naturally want to minimize in the network.

Let $G(N,M)$ be the graph representing a given circuit switched network, where the nodes are the switching stations and the arcs the links between those stations. We now introduce some notation necessary to the model formulation.

Notation

N	: node set for the entire graph;
M	: arc set for the entire graph;
n	: cardinality of N ;
m	: cardinality of M ;
O	: origin node set;
D	: destination node set;
D_p	: destination node set for all flow originating at node p ;
d^{pq}	: demand of origin p destined to node q ;
$\Gamma(i)$	: set of arcs originating at node i ;
$\Gamma^-(i)$	: set of arcs terminating at node i ;
C	: vector representing the network link capacities;
C_u	: capacity of arc u ;
X	: vector representing the network total link flows;
X_u^p	: traffic flow from origin p passing through link u ;
$P_u(X,C)$	: lost call traffic on arc u .

It should be noted that the capacity of any link is given in number of trunks and that the demand parameters and flow variables are measured in Erlangs (average rate of arrival of calls per time unit times average length of calls).

Then, (RM) can be formulated as:

$$\min_X Z = \sum_{u \in M} P_u(X, C) \quad (1)$$

s.t.

Flow conservation

$$\sum_{u \in \Gamma(i)} x_u^p - \sum_{u \in \Gamma^-(i)} x_u^p = \begin{cases} d^{pq} & \text{if } i = p, \\ -d^{pq} & \text{if } i = q, \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

Nonnegativity

$$x_u^p > 0 \quad (3)$$

$P_u(X, C)$, the Erlang expected lost call traffic function, can be written as:

$$P_u(X, C) = \sum_{p \in 0} x_u^p B(C_u, \sum_{p \in 0} x_u^p) \quad (4)$$

where B represents the well known Erlang blocking probability function,

$$B(C_u, x_u) = \frac{x_u^{C_u} / C_u!}{\sum_{i=0}^{\infty} x_u^i / i!} \quad (5)$$

In (5) the variable x_u has replaced the expression $\sum_{p \in 0} x_u^p$ and represents the total offered traffic for link u . $B(C_u, x_u)$ can also be seen as the probability that an arriving call will be blocked given that C_u is the capacity of the link and x_u its total offered traffic. $B(C_u, x_u)$ has been proven to be convex over variable C_u (see [15]), therefore the convexity of $P_u(X, C)$ over C follows. However, $B(C_u, x_u)$ is not convex over variable x_u . Despite this fact, $P_u(X, C)$ has been proven to be convex over variable X (see [16]), a different proof can be found in [13].

For any fixed capacity vector C , (RM) is thus a convex minimization problem which can be solved using standard convex programming techniques.

Network Performance Evaluation

Network reliability has traditionally been defined as a connectivity measure (see [17]-[20]). Since telecommunications networks are well connected and failures are rare, connectivity measures do not necessarily reflect network performance. Furthermore, problems of evaluating many connectivity measures have been proven to be NP-hard. Therefore, in many instances, those connectivity measures cannot be computed except for small networks. Sanso [13] has introduced a new reliability measure which reflects the flow character of the telecommunication network: the Expected Lost Call Traffic (ELCT).

For the sake of simplicity, suppose that only link failures are allowed, that links can be either failed or operating and that failures are independent from one another. Then, if m is the number of links in the network, there are 2^m possible states. Obviously, for any real-sized network it is not practical to study every possible state. However, in [21] and [22], algorithms were developed to partially enumerate only the t most probable states of the system and, in [14], it was shown that for real telecommunication networks, the simple enumeration of single failure and no-failure states will be enough to cover most of the state space probability. Furthermore, Li and Silvester have provided formulas for upper and lower bounds on any network performance measure.

If we denote by S_k the k^{th} state of the system, the upper bound on the ELCT (denoted Z) can be written as:

$$E_{\text{up}} [Z] = \sum_{k=1}^t \Pr(S_k) Z(S_k) + \left(1 - \sum_{k=1}^t \Pr(S_k)\right) Z(S_{2^m}) \quad (6)$$

In expression (6), $P_r(S_k)$ represents the probability of being in state S_k , $Z(S_k)$ represents the LCT for state k and S_{2^m} is the total failure state. Expression (6) can also be interpreted as the sum of two terms; the first one representing the LCT contribution for the t most probable states, and the second one representing a LCT bound for all the other states. In the second term, it is considered that from states $k=t+1$ up to $k=2^m$, all demand will be lost.

It is important to point out that the value of $Z(S_k)$ in (6) corresponds to the final value of the objective function (1) of the routing model, that is:

$$Z(S_k) = \min_X \sum_u P_u(X, C) \quad (7)$$

on the network configuration which exists in state S_k .

3. THE MODEL

The Dimensioning and Routing Model (DRP) can be seen as a problem in which one is to compromise between paying for more installed link capacity or having inferior network performance, especially when failures occur. Formally it can be defined as a network design problem with continuous design variables (see Wong [23]). The model is presented here supposing that the planner has a fairly good idea of the optimal network dimensioning. However, for practical purposes, the algorithm could start with any set of link capacities. We now introduce some notations which will be required to present the model.

Let Q_k be the set defined by constraints (2) and (3) for the routing problem solved for state S_k . Let D_u be the link u capacity already in place, l_u be the cost of adding one unit of capacity to arc u and w the cost of losing one unit of flow over some planning horizon due to link failure or insufficient capacity. Then, problem (DRP) can be stated as:

$$\text{Min}_{X,C} \sum_u 1_u (C_u - D_u) + w E_{up}(Z) \quad (8)$$

s.t.

$$X_u(S_k) \in \Omega_k \quad (9)$$

$$D_u < C_u \quad (10)$$

$$C_u \text{ integer.} \quad (11)$$

Objective function (8) can be simplified by eliminating those terms that do not intervene in the minimization. Hence, problem (DRP) is formulated as follows:

$$\text{Min}_{X,C} \sum_u 1_u C_u + w \sum_{k=1}^t \text{Pr}(S_k) \sum_{u \in M} P_u(X_u(S_k), C_u) \quad (12)$$

s.t. (9) to (11)

The difficulty of solution of problem (DRP) is based on the complexity of function $P_u(X, C)$ (see expressions (4) and (5)). Furthermore, the function presents an asymptotic behavior for large numbers of C which can cause oscillatory problems at the optimization level when X and C are close. Therefore, to ease problem resolution, function P was replaced by an approximation $\hat{P}$. When C is large, it is irrelevant for optimization purposes to restrict C to integer values. The problem can then be re-formulated as

$$(\text{DRP}): \text{Min}_{X,C} \sum_u 1_u C_u + w \sum_{k=1}^t \text{Pr}(S_k) \sum_{u \in M} \hat{P}_u(X_u(S_k), C_u). \quad (13)$$

s.t. (9) to (10)

The Approximation

The first problem encountered to find a good approximation of $P(X,C)$ was to actually evaluate its real value for some given points. In fact, since the link capacity could be in the thousands of trunks a direct evaluation of expression (4) will imply the calculation of more than 1000! To bypass this problem, a computational trick similar to the one shown by Jagerman [24] to compute Erlang's B formula was used. The procedure is shown in the Appendix.

After studying the properties of function $P(X,C)$, it was possible to find an approximation $\hat{P}(X,C)$ as a function of $X-C$ given an interval ϵ .

$$\hat{P}(X,C) = \begin{cases} 0 & \text{if } C - X > \epsilon, \\ \frac{1}{4\epsilon} (X-C+\epsilon)^2 & \text{if } \|X-C\| < \epsilon, \\ X-C & \text{if } X-C > \epsilon. \end{cases} \quad (14)$$

It is straightforward to show that $P(X,C)$ is not only convex over variables X and C , respectively, but is also jointly convex over (X,C) .

To give an idea of the shape of $P(X,C)$ and the goodness of the approximation, both functions have been depicted in figure 1. We point out that a quadratic approximation was chosen when X is close to C to avoid the typical convergence problems at the optimization level, due to the highly asymptotic behavior of function P when X and C are close.

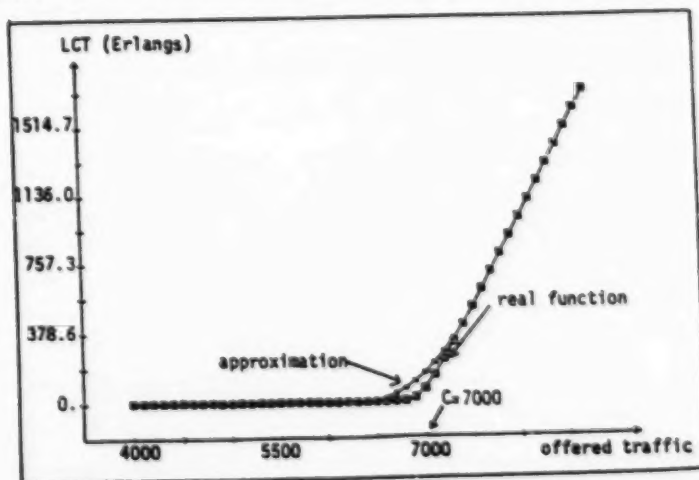


Figure 1 : $P(X,C)$ and $\hat{P}(X,C)$ for $C = 7000$

4. THE ALGORITHM

Problem (DRP) has been solved by a cyclic decomposition algorithm. The general structure of the procedure is shown in figure 2.

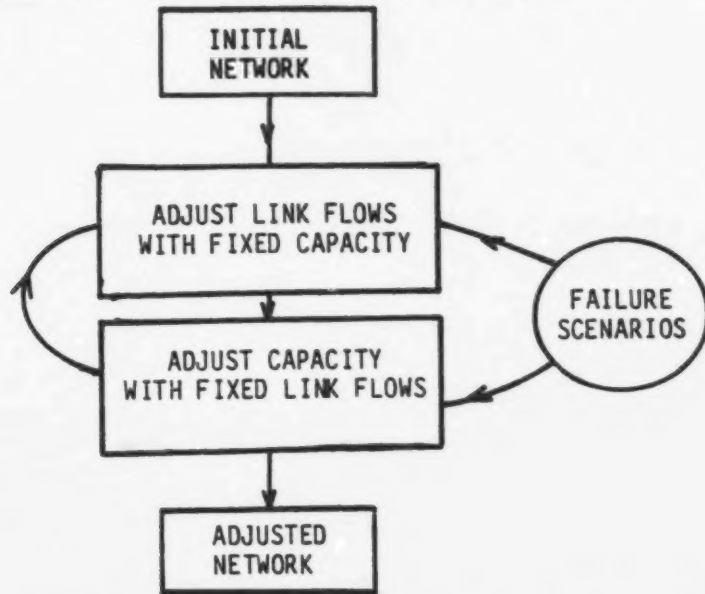


Figure 2: General structure of the cyclic decomposition algorithm

Before getting into the details of the algorithm to solve (DRP) we need to define the dimensioning procedure or problem (DP). This is the problem of finding the best set of link capacities given that link flows have been obtained for every important failure state. Let D represent the set of link capacities already in place, then

$$(DP): \min_{C \geq D} \sum_u l_u C_u + w \sum_{k=1}^t \Pr(S_k) \sum_{u \in m} \hat{P}_u(X_u^*(S_k), C_u) \quad (15)$$

It is easy to see that problem (DP) is separable by arc and that the optimal link capacity is either the solution of

$$\frac{l_u}{w} = - \sum_{k=1}^t \Pr(S_k) \frac{\partial \hat{P}_u}{\partial C_u}(X_u^*(S_k), C_u) \quad (16)$$

or $C_u^* = D_u$ if the solution of (16) is smaller than D_u .

The dimensioning and Routing Algorithm (DRA) can be defined as follow:

STEP 1 : for every state S_k , $k=1, \dots, t$

- solve problem (RM)
- keep the optimum link flows, i.e., $X_u^*(S_k)$ must be stored.

STEP 2 : Solve the dimensioning problem (DP) defined earlier on.

STEP 3 : If the objective function of problem (DRP) remains the same,

STOP, we have found an optimum set (X^*, C^*)
otherwise,
go to STEP 1.

It should be pointed out that even though problem (DRP) can be seen as a variation of the network design with continuous design variables problem defined by Wong, the solution procedure is somewhat more complicated than the ones described in [23]. In fact, the presence of the failure scenarios require us to solve more than one flow problem for a given assignment of link capacities.

The convergence of the (DRA) procedure is easily proven using standard feasible direction results: the objective function is jointly convex in C and X variables; for any S_k , the solution set $X(S_k)$ will be in the compact subset of loopless O/D flows in Q_k ; finally, since the global objective function can only decrease from one iteration to another, the optimal capacity vector will be in the bounded subset τ ,

$$\tau = \{C \in R^m / C \geq D \text{ and } \sum_{u \in M} l_u C_u < Z_1\}$$

where Z_1 is the value of the objective function of (DRP) obtained after first execution of step 1 of the algorithm.

5. COMPUTATIONAL RESULTS

Preliminary results were obtained for a test problem with 10 nodes and 14 two-way links. Failure scenarios corresponded to a mean failure time (MTFy) ranging from 0 to 1500 hours/year. The value of w , the cost of losing one unit of flow, was varied from 50 to 225 times the cost of adding one unit of capacity.

The routing model (RM) was solved with a Frank-Wolfe algorithm whereas the dimensioning procedure (DP) was solved with a line search bisection algorithm. The whole Dimensioning and Routing Algorithm (DRA) was integrated in a C-shell program.

In order to facilitate the convergence of the routing algorithm, the starting feasible solution for iteration i of DRA, $x_u^i(s_k)$ was the optimal solution of the previous iteration, $x_u^{i-1}(s_k)$.

Table I shows some results for $w=225$, $MTFy=600$. The final set of capacities were obtained after 20 iterations.

In Table I the initial set of capacities corresponds to the capacity set "built" with plain common sense. For this set, the objective function value is 889 480 and the cost of capacity is 85 000. The high cost of the objective function comes from the fact that the lost call traffic (LCT) for the no-failure state is very high (3 560).

TABLE I

ARC NUMBER	INITIAL SET OF CAPACITIES	SET OBTAINED AFTER ONE ITERATION	FINAL SET
1	7 000	5 119	5 422
2	10 000	8 022	7 728
3	8 000	4 626	5 150
4	9 000	8 995	10 289
5	10 000	7 495	7 987
6	5 000	4 952	3 944
7	2 000	4 047	3 538
8	4 000	3 984	2 956
9	10 000	7 567	5 619
10	2 000	4 162	4 669
11	6 000	5 990	7 197
12	4 000	3 467	2 941
13	3 000	2 840	3 181
14	5 000	4 044	2 724
TOTAL CAPACITY COST	85 000	75 310	73 345
OBJECTIVE FUNCTION	889 480	86 764	80 497
LCT	3 560	25.7	2.2

After just one iteration of DRA, the objective function descends to 86 764, the cost of capacity installed to 75 310 and the LCT for the no-failure state is 25.70. One iteration of DRA took on the average 40 seconds on a SUN/350.

Finally, the procedure converges to the final set of capacities whose cost is 73 345 and for which the LCT for the no-failure state will be 2.20.

6. CONCLUSIONS AND FURTHER WORK

The routing and dimensioning algorithm has been shown to be a useful tool to adjust link capacities taking into account the possibility of failures and without over dimensioning. The procedure also takes into account the type of routing policy applicable in the network.

We are currently working on some modifications to the procedure that would enable the algorithm to reach the optimal solution more quickly. However, it is worth mentioning that a single iteration of (DRA) already gives the planner an idea on how link capacities must be modified, and this, regardless of the starting set of capacities.

Other interesting extensions of this work would be the application of a similar procedure to packet switched networks and the study of how the dimensioning would change given that traffic patterns are variable or that there are many possible traffic scenarios.

APPENDIX

In this annexe, we show how to evaluate $P(X,C)$ without direct computation of $C!$. To ease the notation, link subindices have been eliminated.

First of all, let us re-write (4) as the inverse of some function $F(X,C)$,

$$P(X,C) = \frac{1}{F(X,C)} \quad (1.1)$$

where

$$F(X,C) = \sum_{i=1}^C \frac{x^i/i!}{x^{C+1}/C!} \quad (1.2)$$

$F(X,C)$ can be re-written as:

$$F(X,C) = \sum_{\tau=0}^C \frac{C!}{(C-\tau)!} \frac{1}{x^{\tau+1}} \quad (1.3)$$

Expression (1.3) is easily reformulated as:

$$F(X,C) = \sum_{j=0}^C A_j \quad (1.4)$$

where A_j is defined as

$$A_j = \begin{cases} \frac{1}{x} & \text{if } j=0, \\ \frac{[C-(j-1)]}{x} A_{j-1} & \text{if } j=1, \dots, C. \end{cases} \quad (1.5)$$

The iterative expression (1.5) allows us to exactly evaluate $P(X,C)$ just by coding a loop in a simple program while maintaining a high level of numerical accuracy.

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